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Karimian-Azari, Sam; Mohammadiha, Nasser; Jensen, Jesper Rindom; Christensen, Mads Græsbøll

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Pitch Estimation and Tracking with Harmonic Emphasis on the Acoustic Spectrum

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Sam Karimian-Azari[1], Nasser Mohammadiha[2],
Jesper R. Jensen[1], and Mads G. Christensen[1]

ska@create.aau.dk

[1]Audio Analysis Lab, AD:MT, Aalborg University

[2]University of Oldenburg

Agenda

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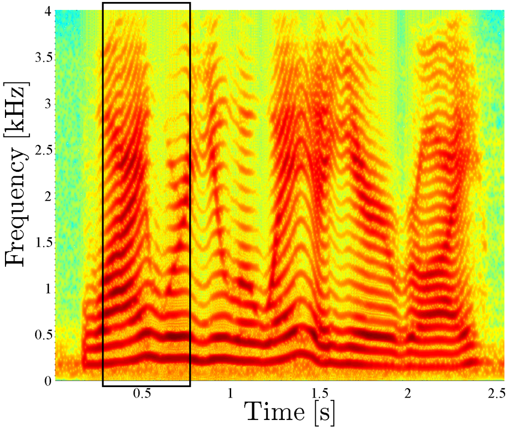
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3
2.5
2
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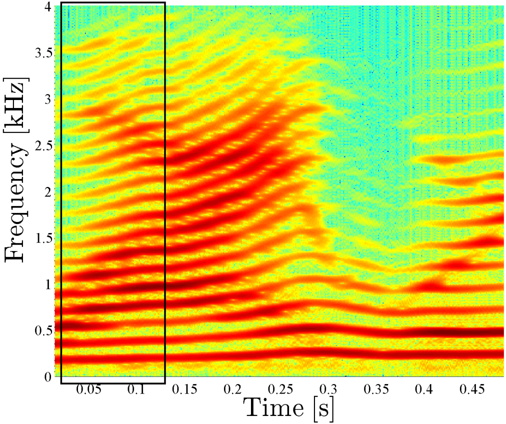
1
0.5
0
0.5
1
1.5
2
2.5
Time [s]



Frequency [kHz]
Time [s]
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0.5
1
1.5
2
2.5
3
3.5
4
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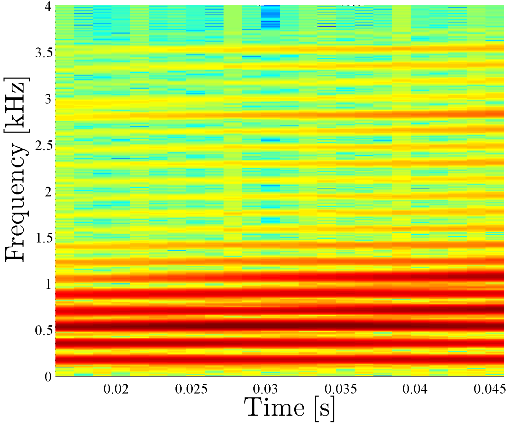
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

Introduction
Harmonic Signal Model



Harmonic Signal Model:

$$s(n) = \sum_{l=1}^{L(n)} \alpha_l \, e^{j\,(\omega_l(n)\, n + \varphi_l)}, \qquad (1)$$

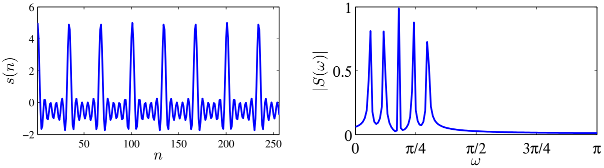
where $\omega_l(n) = l\omega_0(n)$ for $l = 1, \ldots, L(n)$,

$L(n)$: number of sinusoids

α_l : real magnitudes

ω_0 : fundamental frequency

φ_l : phases of harmonics



Signal Model
Additive noise

The observed signal can be written as a sum of a desired signal $s(n)$ and a noise signal $v(n)$, i.e.,

$$\begin{aligned} x(n) &= s(n) + v(n) \\ &= \sum_{l=1}^{L} \alpha_l \, e^{j\,(\omega_l n + \varphi_l)} + v(n). \end{aligned} \tag{2}$$

At a high narrowband SNR, the harmonic frequency ω_l is perturbed with a real-valued phase-noise [S.Tretter 1985], which has a normal distribution with zero mean and the variance

$$\mathrm{E}\{\Delta\omega_l^2(n)\} = \frac{\sigma^2}{2\alpha_l^2} \tag{3}$$

We can approximate $x(n) = \sum_{l=1}^{L} \alpha_l \, e^{j\,(\omega_l n + \varphi_l)} + v(n)$ like

$$x(n) \approx \sum_{l=1}^{L} \alpha_l \, e^{j\,(\omega_l n + \Delta\omega_l(n) + \varphi_l)} \tag{4}$$

Unconstrained frequency estimates (UFE) of the constrained frequencies:

$$\hat{\boldsymbol{\Omega}}(n) = \left[\,\hat{\omega}_1(n),\, \hat{\omega}_2(n),\, \ldots,\, \hat{\omega}_L(n)\,\right]^T \tag{5}$$

$$= \mathbf{d}_L(n)\,\omega_0(n) + \Delta\boldsymbol{\Omega}(n), \tag{6}$$

where

$$\mathbf{d}_L(n) = \left[\,1, 2,\, \ldots, L(n)\,\right]^T \tag{7}$$

$$\Delta\boldsymbol{\Omega}(n), = \left[\,\Delta\omega_1(n),\, \Delta\omega_2(n),\, \ldots,\, \Delta\omega_L(n)\,\right]^T, \tag{8}$$

and

$$\boldsymbol{R}_{\Delta\Omega}(n) = \mathrm{E}\{\Delta\boldsymbol{\Omega}(n)\Delta\boldsymbol{\Omega}^T(n)\} \tag{9}$$

$$= \frac{\sigma^2}{2}\,\mathrm{diag}\left\{\frac{1}{\alpha_1^2}, \frac{1}{\alpha_2^2}, \ldots, \frac{1}{\alpha_L^2}\right\}.$$

Max. Likelihood (ML) Pitch Estimator

For the time-frame $\mathbf{x}(n) = \left[\, x(n), x(n-1), \ldots, x(n-M-1) \,\right]^T$, the PDF of the UFE is

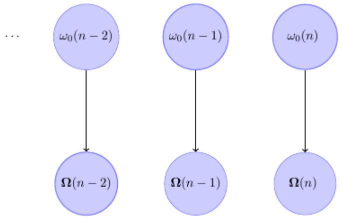
$$P(\hat{\boldsymbol{\Omega}}(n)|\omega_0(n)) \sim \mathcal{N}(\mathbf{d}_L(n)\,\omega_0(n), \boldsymbol{R}_{\Delta\Omega}(n)). \tag{10}$$

The ML pitch estimator:

$$\hat{\omega}_0(n) = \arg\max_{\omega_0(n)} \log P(\hat{\boldsymbol{\Omega}}(n)|\omega_0(n)) \tag{11}$$

$$= \left[\, \mathbf{d}_L^T(n) \boldsymbol{R}_{\Delta\Omega}^{-1}(n) \mathbf{d}_L(n) \,\right]^{-1} \mathbf{d}_L^T(n) \boldsymbol{R}_{\Delta\Omega}^{-1}(n) \hat{\boldsymbol{\Omega}}(n) \tag{12}$$

- The ML Estimators are statistically efficient, e.g., the non-linear least-squares (NLS), and the weighted least squares (WLS) [H.Li, et al. 2000], but the minimum variance is limited by the number of samples.
- Consecutive pitch values are estimated independently

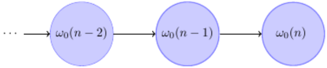


Bayesian Pitch Estimator
Motivation

- Pitch values are usually correlated in a sequence, i.e.,

$$P(\omega_0(n)|\omega_0(n-1), \omega_0(n-2), \cdots), \tag{13}$$

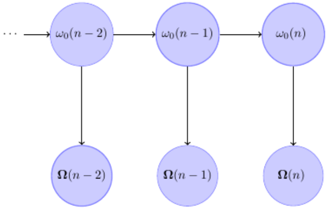
  that motivate Bayesian methods to minimize an error incorporating prior distributions.
- State-of-the-art methods mostly track pitch estimates in a sequential process without concerning noise statistics.



1- Jointly estimate and track pitch incorporating both the harmonic constraints and noise characteristics.
2- Estimate the state $\omega_0(n)$ through a series of noisy observations:

$$P(\omega_0(n)|\hat{\mathbf{\Omega}}(n), \hat{\mathbf{\Omega}}(n-1), \cdots) \tag{14}$$

3- Recursively update the prior distribution of the pitch value.



Bayesian Pitch Estimator

Discrete state-space (HMM)

$\omega_0(n)$: Discrete random variable (Hidden states)

$P(\omega_0(n)|\omega_0(n-1))$: Transition probability in a 1st-order Markov model,

$$\text{i.e.,} \sum_{\omega_0(n)} P(\omega_0(n)|\omega_0(n-1)) = 1$$

$$\begin{aligned}\hat{\omega}_0(n) &= \arg\max_{\omega_0(n)} \log P(\omega_0(n)|\hat{\boldsymbol{\Omega}}(n), \hat{\boldsymbol{\Omega}}(n-1), \cdots) \\ &= \arg\max_{\omega_0(n)} \log P(\hat{\boldsymbol{\Omega}}(n)|\omega_0(n)) + \log P(\omega_0(n)|\hat{\boldsymbol{\Omega}}(n-1), \cdots).\end{aligned} \tag{15}$$

The priori distribution is defined recursively like

$$\begin{aligned}&P(\omega_0(n)|\hat{\boldsymbol{\Omega}}(n-1), \hat{\boldsymbol{\Omega}}(n-2), \cdots) = \\ &\sum_{\omega_0(n-1)} P(\omega_0(n)|\omega_0(n-1)) P(\omega_0(n-1)|\hat{\boldsymbol{\Omega}}(n-1), \cdots),\end{aligned} \tag{16}$$

where $P(\omega_0(n-1)|\hat{\boldsymbol{\Omega}}(n-1), \cdots)$ is the past estimate.

Continuous state-space:

$$\omega_0(n) = \omega_0(n-1) + \delta(n)$$
$$\hat{\boldsymbol{\Omega}}(n) = \mathbf{d}_L(n)\,\omega_0(n) + \Delta\boldsymbol{\Omega}(n),$$

where $\delta(n) \sim \mathcal{N}(0, \sigma_t^2)$ and $\Delta\boldsymbol{\Omega}(n) \sim \mathcal{N}(\mathbf{0}, \boldsymbol{R}_{\Delta\Omega}(n))$ are the state evolution and observation noise, respectively.

Bayesian Pitch Estimator

Continuous state-space (Kalman filter)

First, a pitch estimate is predicted using the past estimates as

$$\hat{\omega}_0(n|n-1) = \hat{\omega}_0(n-1|n-1) \tag{17}$$

with the variance

$$\sigma^2_{\mathrm{K}}(n|n-1) = \sigma^2_{\mathrm{K}}(n-1|n-1) + \sigma^2_t. \tag{18}$$

Second, the pitch estimate is updated with the error of

$$\mathbf{e}(n) = \hat{\boldsymbol{\Omega}}(n) - \mathbf{d}_L(n)\,\hat{\omega}_0(n|n-1). \tag{19}$$

Then, the predicted estimate is updated:

$$\hat{\omega}_0(n|n) = \hat{\omega}_0(n|n-1) + \mathbf{h}_{\mathrm{K}}(n)\mathbf{e}(n) \tag{20}$$

$$\mathbf{h}_{\mathrm{K}}(n) = \sigma^2_{\mathrm{K}}(n|n-1)\mathbf{d}_L^T(n)\Big[\boldsymbol{\Pi}_L(n)\sigma^2_{\mathrm{K}}(n|n-1) + \boldsymbol{R}_{\Delta\Omega}(n)\Big]^{-1}, \tag{21}$$

where $\boldsymbol{\Pi}_L(n) = \mathbf{d}_L(n)\mathbf{d}_L^T(n)$, and update

$$\sigma^2_{\mathrm{K}}(n|n) = \Big[1 - \mathbf{h}_{\mathrm{K}}(n)\mathbf{d}_L(n)\Big]\sigma^2_{\mathrm{K}}(n|n-1). \tag{22}$$

The ML estimator of the covariance matrix among N estimates:

$$\mathbf{R}_{\Delta\boldsymbol{\Omega}}(n) = \mathrm{E}\{\Delta\boldsymbol{\Omega}(n)\Delta\boldsymbol{\Omega}^T(n)\} = \frac{1}{N}\sum_{i=n-N+1}^{n} \Delta\boldsymbol{\Omega}(i)\Delta\boldsymbol{\Omega}^T(i), \tag{23}$$

where $\Delta\boldsymbol{\Omega}(n) = \hat{\boldsymbol{\Omega}}(n) - \hat{\boldsymbol{\mu}}(n)$, and $\boldsymbol{\mu}(n) = \mathrm{E}\{\hat{\boldsymbol{\Omega}}(n)\}$.

Exponential moving average:

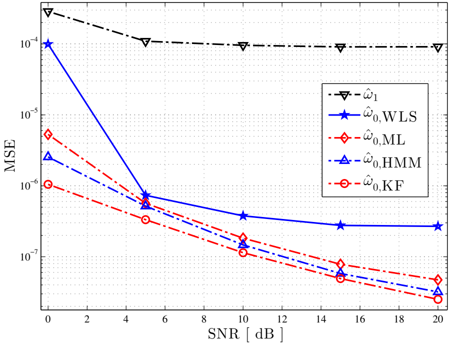
$$\hat{\boldsymbol{\mu}}(n) = \lambda\,\hat{\boldsymbol{\Omega}}(n) + (1-\lambda)\,\hat{\boldsymbol{\mu}}(n-1) \tag{24}$$

The forgetting factor $0 < \lambda < 1$ recursively updates the time-varying mean value.

Numerical Results

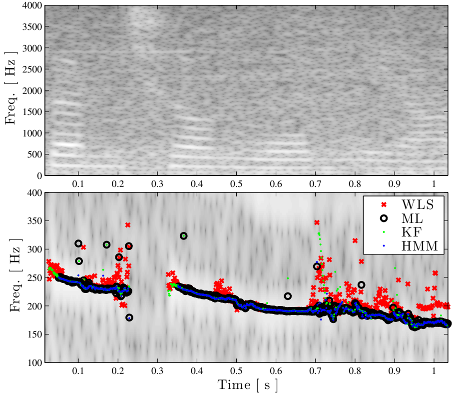
Synthetic signal

A linear chirp signal ($r = 100$ Hz/s) with $L = 5$ harmonics, random phases, and identical amplitudes during 0.1 s.



$M = 80$, $\omega_0(1) = 400\pi/f_s$, $fs = 8.0$ kHz, $\sigma_t = \sqrt{2\pi r/f_s^2}$, and for the HMM-based pitch estimator, the frequency range $\omega \in [150, 280] \times (2\pi/f_s)$ was discretized into $N_d = 1000$ samples.

Speech signal + Car noise at SNR= 5 dB.



The MAP order estimation [Djuric 1998], $M = 240$, $\lambda = 0.9$, and $N = 150$.

Conclusion

- For pitch estimation, we have formulated the ML estimate from the UFE.
- For pitch estimation and tracking, we have proposed HMM- and KF-based methods.
- Experimental results showed that both HMM- and KF-based methods outperform the corresponding ML pitch estimators.
- The KF-based method statistically performs better than the HMM-based method, while the it tracks pitch changes more accurate than the KF-based method.

Thank you!

AALBORG UNIVERSITY
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